Configuration of a Towline Attached to a Vehicle Moving in a Circular Path

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The problem of predicting the steady-state performance of a cable-body system towed in a circular path is studied analytically. Equations governing the steady-state configuration of the towline, distribution of the tension along the towline, and the location of the towed body are derived. Hydrodynamic forces accounted for in the analysis are the buoyancy, the side force, the added-mass effect as well as the dominant fluid resistance. The present model can be applied to both aerial and underwater towed systems. The analytical model is evaluated by comparison of its predictions with limited data of laboratory tests. Numerical solutions are obtained for the case of underwater towing of spherical bodies. Based on these results, some remarks on the operational aspects of the underwater towing are presented.

Nomenclature

A = cross-sectional area of towline
A_B = frontal area of towed sphere, on which C_{DB} is based
$\mathbf{b}$ = $\mathbf{V} \times \boldsymbol{\tau}/(V \sin\phi)$, Fig. 2
C_D = drag coefficient, $D/(\frac{1}{2}\rho V^2 L)$ where D is the total drag per unit length of a cylinder placed normal to a stream
C_{DB} = drag coefficient of towed sphere
d = diameter of circular or stranded cable
$\mathbf{e}_R, \mathbf{e}_\theta$ = unit vectors in the R-, θ-directions
$\mathbf{F}_B$ = sum of all the external forces acting on a towed body, except $T_0\boldsymbol{\tau}_0$
$\mathbf{f}_a, f_n, f_\tau, f_s$ = contribution to the total hydrodynamic force per unit length of towline due respectively to added-mass effect, normal resistance, tangential resistance, and side force
g = acceleration due to gravity
$\mathbf{I},\mathbf{J},\mathbf{K}$ = unit vectors in the X-, Y-, Z-directions, Fig. 1
K_s = an experimental constant for the side force of a stranded cable, Eq. (28)
ℓ = total length of towline ($s = \ell$ at the point of attachment to the towing ship)
M_B = virtual mass of towed sphere, Eq. (43)
m = mass per unit length of towline at s (may vary along s)
m_B = mass of towed body
$\mathbf{n}$ = $\mathbf{b} \times \boldsymbol{\tau}$, Fig. 2
P = a material point on the axis of towline
R,θ,Y = circular cylindrical coordinates rotating with the ship about the Y-axis, Fig. 1
$\mathbf{r}$ = position vector of a material point P on the towline, Fig. 1
Re = $d\rho V/\mu$, Reynolds number
Re_n = $d\rho V_n/\mu = Re\cdot\sin\phi$
s = material coordinate, representing the arc length between the body end and a material point P of towline, Fig. 1
T = magnitude of the tension in towline at $P(s,t)$
t = time
V = $|\mathbf{V}|$, magnitude of $\mathbf{V}$
$\mathbf{V}$ = velocity of towline at $P(s,t)$ with respect to the inertial coordinates
$\bar{V}_B$ = volume of towed sphere
w = net weight of towline in fluid per unit length, Eq. (13)
w_B = net weight of towed sphere in fluid, Eq. (44)
X,Y,Z = Cartesian coordinates rotating about the Y-axis with the towing ship, Fig. 1
x,y,z = inertial coordinates, with y representing elevation
μ = coefficient of viscosity of fluid
ρ = density of fluid
$\boldsymbol{\tau}$ = unit vector tangent to the axis of towline at $P(s,t)$ and in the direction of increasing s, Fig. 2
ϕ = angle subtended by $\boldsymbol{\tau}$ and V, Fig. 2
ψ = angle subtended by the R-axis and the projection of $\boldsymbol{\tau}$ onto the RY-plane, Fig. 2
Ω = angular speed of the towing ship, the towed system and the rotating coordinates X,Y,Z and R,θ,Y
$\boldsymbol{\Omega}$ = $\Omega\mathbf{J}$

Subscripts

0 = value at $s = 0$
ℓ = value at $s = \ell$

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Introduction

THE problem of predicting the steady-state performance of a cable-body system towed in a circular path has recently encountered practical importance. One feature of this type of towing is of particular interest, that is, under certain conditions, the towed body obtains an equilibrium position very near the axis of rotation. This phenomenon has been exploited for the pinpoint delivery of a payload, the search of a particular area, and the surface-to-air retrieval of an object such as a spacecraft. Essentially similar technology is involved in the sonar-lowering from a fixed-wing aircraft orbiting in a circular path, and orbiting antennas for airborne VLF communications.

Previous investigations by Kline[1] and Huang[2] produced equilibrium configuration of a cable towed by an orbiting aircraft. They treated the cable as continuum. Skop[3] performed the same kind of analysis and obtained solutions for vacuous towing media. Huang and Skop reported independently that multiple steady-state solutions can exist under certain combinations of governing parameters. Skop and Choo[4] investigated the modelling laws for steady, circular towing, and studied further the multiplicity of the solutions for aerial systems. Crist[5] developed a lumped-mass model

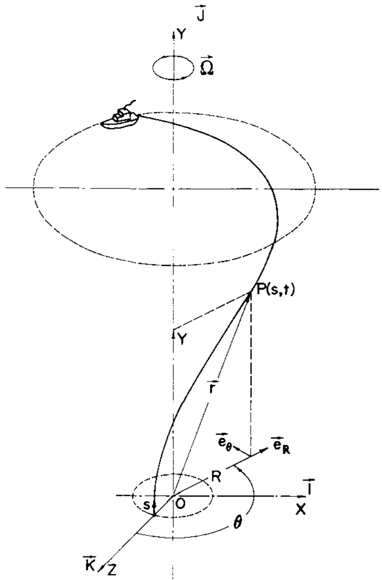

Fig. 1 The towed system and the rotating coordinates XYZ and $R\Theta Y$.

and studied the effect of aircraft oscillations and the behavior of the cable during the transition from orbit to straight-and-level flight.

The previous investigators were essentially interested in aerial towed systems. Consequently, the prior works did not include the buoyancy, the added-mass effect, and the side force. However, in the underwater environment, these forces are not so small as to be neglected. In this study that includes all of these forces, a method of obtaining the steady-state configuration of a cable towed in a circular path is developed which can be used for underwater towed systems as well as aerial towed systems. Some computational results are obtained for underwater towing of spherical bodies, and some remarks on their operational aspects are presented.

Analytical Solution of the Problem

Formulation of the Governing Equations

See Fig. 1 which shows a towed system that moves steadily about a fixed axis. Let us fix Cartesian coordinates x,y,z in the ocean such that the y-axis coincides with the vertical axis about which the towing ship moves. These coordinates are referred to as inertial coordinates since the effect of the Earth's rotation is neglected in this report. Let X,Y,Z be Cartesian coordinates rotating with the ship about the Y-axis. Thus, the Y-axis coincides with the y-axis of the inertial coordinates. Furthermore, we introduce circular-cylindrical coordinates R,θ,Y, rotating with the ship about the Y-axis. Origins of these three coordinate systems are all located at the point of intersection of the Y-axis and the horizontal plane containing the lower end of the cable. Let s denote the arc length measured from the cable-attachment point of the towed body to a material point P on the axis of the towline. Then, all the variables can be regarded as functions of s and t in the inertial coordinates (x,y,z), but functions of only s in the rotating coordinates $(X,Y,Z;\ R,\theta,Y)$.

The vectors $\mathbf{I},\mathbf{J},\mathbf{K}$ and $\mathbf{e}_R,\mathbf{e}_\theta$ are unit vectors in the directions, respectively, of X,Y,Z and R,θ. In general, the orientations of $\mathbf{e}_R$ and $\mathbf{e}_\theta$ vary with s along the towline. Then, the position $\mathbf{r}$ of a material point P on the axis of the towline is given by

$$\mathbf{r} = X\mathbf{I} + Y\mathbf{J} + Z\mathbf{K} = R\mathbf{e}_R + Y\mathbf{J} \tag{1}$$

To decompose the external forces acting on a towline element in a more meaningful way, we introduce the natural coordinates $\boldsymbol{\tau},\mathbf{n},\mathbf{b}$. The vectors are formed in the following manner (Fig. 2). $\boldsymbol{\tau}$ is defined as a unit vector tangent to the axis of the cable at $P(s,t)$ and in the direction of increasing s. Thus, $\boldsymbol{\tau} \equiv \partial\mathbf{r}/\partial s$ for an inextensible towline. Let $\mathbf{V}$ be the velocity of the point P with respect to the inertial coordinates, and V be $|\mathbf{V}|$, that is, the magnitude of $\mathbf{V}$. Let ϕ denote the angle subtended by $\boldsymbol{\tau}$ and $\mathbf{V}$. This angle is measured counterclockwise from the $\mathbf{V}$-vector to the $\boldsymbol{\tau}$-vector. Then, $\mathbf{b} \equiv \mathbf{V} \times \boldsymbol{\tau}/(V \sin\phi)$ is a unit vector normal to both $\boldsymbol{\tau}$ and $\mathbf{V}$, and $\mathbf{n} \equiv \mathbf{b} \times \boldsymbol{\tau}$ is a unit vector that is normal to $\boldsymbol{\tau}$, and lies in the plane containing $\boldsymbol{\tau}$ and $\mathbf{V}$. Thus, these three unit vectors $\boldsymbol{\tau}$, $\mathbf{n}$, and $\mathbf{b}$ form a mutually orthogonal, right-handed set of local coordinate vectors.

The vector angular velocity of the towing ship, the towed system, and the rotating coordinates is given by

$$\boldsymbol{\Omega} = \Omega\mathbf{J} \tag{2}$$

where the angular speed Ω is positive when the axis of rotation remains to the port of the ship as in Fig. 1. The velocity and the acceleration of the towline are

$$\mathbf{V} = \boldsymbol{\Omega} \times \mathbf{r} = \Omega R\mathbf{e}_\theta \tag{3}$$

$$\partial\mathbf{V}/\partial t = \boldsymbol{\Omega} \times \mathbf{V} = -\Omega^2 R\mathbf{e}_R \tag{4}$$

Relations between the natural and the cylindrical coordinates can be found from the definitions of these coordinates and their geometry (Fig. 2). These are

$$\boldsymbol{\tau} = \mathbf{e}_\theta \cos\phi + \mathbf{J} \sin\psi \sin\phi + \mathbf{e}_R \cos\psi \sin\phi \tag{5a}$$

$$\mathbf{n} = -\mathbf{e}_\theta \sin\phi + \mathbf{J} \sin\psi \cos\phi + \mathbf{e}_R \cos\psi \cos\phi \tag{5b}$$

$$\mathbf{b} = -\mathbf{J} \cos\psi + \mathbf{e}_R \sin\psi \tag{5c}$$

and

$$\begin{aligned} \mathbf{e}_R &= \boldsymbol{\tau} \cos\psi \sin\phi + \mathbf{b} \sin\psi + \mathbf{n} \cos\psi \cos\phi \\ \mathbf{e}_\theta &= \boldsymbol{\tau} \cos\phi - \mathbf{n} \sin\phi \\ \mathbf{J} &= \boldsymbol{\tau} \sin\psi \sin\phi - \mathbf{b} \cos\psi + \mathbf{n} \sin\psi \cos\phi \end{aligned} \tag{6}$$

Differentiating Eq. (1) with respect to s, and using

$$d\mathbf{e}_R/d\theta \equiv \mathbf{e}_\theta; \qquad d\mathbf{e}_\theta/d\theta \equiv -\mathbf{e}_R, \tag{7}$$

we obtain

$$\boldsymbol{\tau} \equiv d\mathbf{r}/ds = \mathbf{e}_R dR/ds + \mathbf{e}_\theta R d\theta/ds + \mathbf{J} dY/ds \tag{8}$$

Comparing Eq. (8) with Eq. (5a), we get

$$dR/ds = \cos\psi \sin\phi \tag{9}$$

$$d\theta/ds = \cos\phi/R \tag{10}$$

$$dY/ds = \sin\psi \sin\phi \tag{11}$$

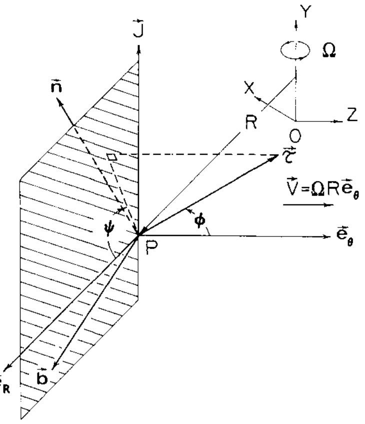

Fig. 2 Schematic diagram illustrating the local natural coordinate system for a towline and its relation to a rotating coordinate system $R\Theta Y$.

Differentiating Eq. (5a) with respect to s, and using Eq. (6), we find

$$d\boldsymbol{\tau}/ds = -\mathbf{b}[\sin\phi d\psi/ds + (1/R)\sin\psi\cos^2\phi] + \mathbf{n}[d\phi/ds - (1/R)\cos\psi\cos\phi] \quad (12)$$

We have derived some geometrical relations. Now, let us derive equations of motion of a towline, assuming that it is an inextensible, circular cylindrical or stranded cable of zero bending rigidity.

Let m denote mass per unit length of towline, T magnitude of the tension in towline at $P(s,t)$, and w net weight of towline in fluid per unit length, that is,

$$w \equiv mg - \rho g A \quad (13)$$

In Eq. (13) A is cross-sectional area of towline, and $-\rho g A$ is buoyancy. Then, equation of motion of a towline in inertial coordinates is[6]

$$m\partial\mathbf{V}/\partial t = (\partial/\partial s)(T\boldsymbol{\tau}) - w\mathbf{j} + \mathbf{n}f_n + \boldsymbol{\tau}f_\tau + \mathbf{b}f_s + \mathbf{f}_a \quad (14)$$

where f_n is the normal resistance, $\mathbf{f}_\tau$ the tangential resistance, f_s the side force, and $\mathbf{f}_a$ the force due to added-mass effect, all per unit length of towline. Expressions of these hydrodynamic forces shall be given in the following subsection.

Noting that

$$(\partial/\partial s)(T\boldsymbol{\tau}) = T\partial\boldsymbol{\tau}/\partial s + \boldsymbol{\tau}\partial T/\partial s \quad (15)$$

and substituting Eqs. (4) and (12) into Eq. (14), we obtain

$\boldsymbol{\tau}$-direction:

$$dT/ds = w\sin\psi\sin\phi - m\Omega^2 R\cos\psi\sin\phi - f_\tau - \mathbf{f}_a\cdot\boldsymbol{\tau} \quad (16)$$

$\mathbf{n}$-direction:

$$Td\phi/ds = w\sin\psi\cos\phi - (m\Omega^2 R - T/R)\cos\psi\cos\phi - f_n - \mathbf{f}_a\cdot\mathbf{n} \quad (17)$$

$\mathbf{b}$-direction:

$$T\sin\phi d\psi/ds = w\cos\psi + [m\Omega^2 R - (T/R)\cos^2\phi]\sin\psi + f_s + \mathbf{f}_a\cdot\mathbf{b} \quad (18)$$

Thus, the tangential components of the external forces act to increase the tension, while the normal and the side components cause the towline to bend, changing the line of action but not directly affecting the magnitude of the tension.

Hydrodynamic Forces on Towlines

Expression of each of the hydrodynamic forces f_n, f_τ, f_s, and $\mathbf{f}_a$ are given in this subsection. For details of derivation of these formulas, see Ref. 6. Reynolds number (Re), an important parameter of hydrodynamic similarity, is defined as

$$Re \equiv d\rho V/\mu \quad (19)$$

where d is the diameter of the cable, ρ the density of the fluid, and μ the coefficient of viscosity of fluid. In cable hydrodynamics, Re_n, the Reynolds number based on the velocity component perpendicular to the axis of the cable (V_n), is often used. Thus,

$$Re_n \equiv d\rho V_n/\mu = Re\cdot\sin\phi \quad (20)$$

where

$$V_n = V\sin\phi \qquad V_\tau = V\cos\phi \quad (21)$$

The values of Reynolds numbers encountered in towing operations lie between 10^3 and 10^5.

Hydrodynamic resistance or drag of a cable is caused by pressure imbalance fore and aft of the cable and fluid friction over the cable surface. Conventionally, the total resistance is resolved into the normal resistance ($\mathbf{n}f_n$) and the tangential resistance ($\boldsymbol{\tau}f_\tau$), respectively normal and tangential to the cable axis. Formula of f_n for circular cables is given by

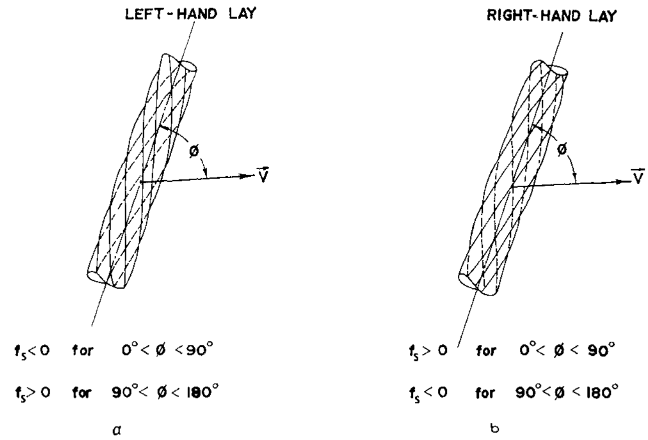

Fig. 3 Side force of stranded cables.

$$f_n = C_D(Re_n)d\tfrac{1}{2}\rho V_n^2 \quad (22)$$

with

$$C_D = [8\Pi/(Re_n S)](1 - 0.87S^{-2}) \qquad (0 < Re_n \le 1) \quad (23)$$

where

$$S = -0.077215665 + \ln(8Re_n^{-1}) \quad (24)$$

and

$$C_D = 1.45 + 8.55\,Re_n^{-0.90} \qquad (1 < Re_n < 30) \quad (25)$$

$$C_D = 1.1 + 4Re_n^{-1/2} \qquad (30 \le Re_n < 10^5) \quad (26)$$

In Eq. (22), $C_D(Re_n)$ means that the drag coefficient C_D is a function of Re_n. Formula of f_τ for circular cables is

$$f_\tau = -\Pi\mu V_\tau\,(0.55\,Re_n^{1/2} + 0.084\,Re_n^{2/3}) \qquad (\phi \neq 0) \quad (27)$$

Equations (25–27) are based on experimental data. Since available experimental data[7–12] for stranded cables are widely scattered and do not exhibit any discernible trend, we propose to use Eqs. (22–27) also for stranded cables.

A stranded cable in a stream experiences a side force, that is, a force in the $+\mathbf{b}$- or $-\mathbf{b}$-direction (see Fig. 3). Owing to the lay of the strands and the general inclination of the towed cable, there is a tendency towards tangential flow down the strands on one side of the cable (in Fig. 3a, the side on which the lays appear as dotted lines) and normal flow across the strands on the opposite side. Thus, the fluid flow is unsymmetrical about the $\boldsymbol{\tau}\mathbf{n}$-plane, and this results in the side force. According to Gay,[13]

$$f_s = \pm(K_s\cos^{1/2}\phi\sin\phi Re^{-1/2})d\tfrac{1}{2}\rho V^2 \quad (28)$$

where the algebraic sign is determined according to Fig. 3, and K_s is an experimental constant representative of the degree of unsymmetry of the flow. Of course, the value of K_s depends on the type of the stranded cable. Again, due to wide scatter of existing data,[10–13] accurate values of K_s cannot be determined; $K_s = 10$ was chosen conservatively.

We have seen that there exists an urgent need for systematic measurements of the hydrodynamic forces acting on stranded cables. Added-mass effect is explained on the basis of the potential-flow theory. Any motion of a body induces a motion in the otherwise stationary fluid because the fluid must move aside and then close in behind the body in order that the body may make a passage through the fluid. As a consequence, the fluid possesses kinetic energy that it would lack if the body were not in motion. The body had to impart the kinetic energy to the fluid by doing work on the fluid, that is, the work done is equal to the change in kinetic energy. The fluid disturbance resulting from a moving body extends

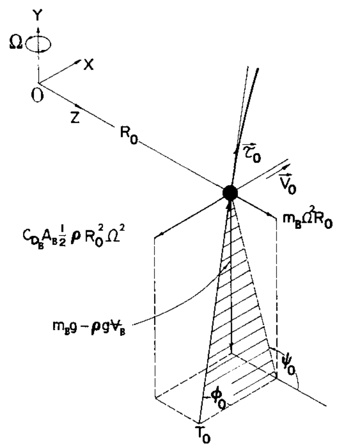

Fig. 4 Forces acting on a towed sphere.

in decreasing amplitude to infinity, but from a body dynamic standpoint this may be treated as localized to a finite volume of fluid. The body behaves as though an added mass of fluid were moving with it. When the motion of the body is steady, the corresponding fluid motion is steady and the kinetic energy in the fluid is constant. Hence no work is done on the fluid so long as the motion of the body remains steady. It follows that the added-mass terms must be included in the equations of motion only when nonsteady motion is studied.

In steady turn, the towline is continually accelerated towards the axis of rotation. Therefore, the added-mass effect must be considered in our problem. From potential-flow theory, only the acceleration perpendicular to cable axis affects the fluid, and the added mass for a circular cable is ρA per unit length.[14] We assume that the added mass of a stranded cable is also ρA. Then, we have

$$\mathbf{f}_a = \mathbf{b}\rho A \cdot R\Omega^2 \sin\psi \sin^2\phi \qquad (29)$$

Summary of Governing Equations

Now that we have the formulas of the hydrodynamic forces, Eqs. (22–29), we substitute these equations into Eqs. (16–18) to obtain the working equations of motion

$$dT/ds = w \sin\psi \sin\phi - m\Omega^2 R \cos\psi \sin\phi + \pi\mu V_\tau(0.55\, Re_n^{1/2} + 0.084\, Re_n^{2/3}) \qquad (30)$$

$$d\phi/ds = (1/T)[w \sin\psi \cos\phi - (m\Omega^2 R - T/R) \cos\psi \cos\phi - C_D(Re_n)d\tfrac{1}{2}\rho V_n^2 \qquad (31)$$

$$d\psi/ds = (1/T \sin\phi)\{w \cos\phi + [\Omega^2 R(m + \rho A \sin^2\phi) - (T/R)\cos^2\phi] \sin\psi \pm K_s \cos^{1/2}\phi \sin\phi Re^{-1/2})d\tfrac{1}{2}\rho V^2\} \qquad (32)$$

$C_D(Re_n)$ in Eq. (31) is given by Eqs. (23–26). In Eq. (32), K_s is zero for circular cables, and is roughly equal to 10 for stranded cables; the algebraic sign is determined according to Fig. 3. Geometrical relations to be satisfied are Eqs. (9–11), that is,

$$dR/ds = \cos\psi \sin\phi \qquad (33)$$

$$d\theta/ds = \cos\phi/R \qquad (34)$$

$$dY/ds = \sin\psi \sin\phi \qquad (35)$$

To sum up, the problem of steady circular towing involves six nonlinear, ordinary differential equations (30–35) relating the six variables T,ϕ,ψ,R,θ,Y.

The conditions to be satisfied at the upper and lower ends are specified in the following manner. We assume that, at the upper end ($s = \ell$), the turning radius and speed of the towing vehicle are given. These two values determine the angular speed Ω. The conditions to be satisfied at the lower end ($s = 0$) are the equation of motion of the towed body and geometrical relations showing the location of the lower end. The former is

$$-\mathbf{e}_R m_B R_0 \Omega^2 = T_0 \boldsymbol{\tau}_0 + \mathbf{F}_B \qquad \text{at } s = 0 \qquad (36)$$

where m_B is mass of the towed body, and $\mathbf{F}_B$ is vector sum of the forces acting on the body, except the cable tension. In general, $\mathbf{F}_B$ consists of its weight, buoyancy, added-mass effect, downward lift of the depressing wing, and hydrodynamic side force due to unsymmetry of the body. Equation (36) yields the magnitude of T_0 and its orientation. Without loss of generality, we can say that the location of the lower end is

$$R = R_0 \qquad \text{at} \qquad s = 0 \qquad (37)$$

$$\theta = 0 \qquad \text{at} \qquad s = 0 \qquad (38)$$

$$Y = 0 \qquad \text{at} \qquad s = 0 \qquad (39)$$

Numerical Method of Solution

Since the value of R_0 is not initially known for any given problem, and since the integration of the equations must proceed from $s = 0$, we have to assume a value of R_0 and integrate the differential equations. If the value of R_ℓ resulting from the integration is different from the specified value, we assume another value of R_0, and repeat the same procedure until the difference between the calculated and the specified values of R_ℓ vanishes. Note that geometrical considerations bound the possible values of R_0 to the range

$$0 < R_0 \leq R_\ell + \ell \qquad \text{for} \qquad \ell \geq R_\ell \qquad (40)$$

or

$$R_\ell - \ell \leq R_0 \leq R_\ell + \ell \qquad \text{for} \qquad \ell < R_\ell \qquad (41)$$

A computer program has been developed that integrates the six differential equations using the Runge-Kutta method. For the details of the computer program, see Ref. 6. The step size of numerical integration is changed such that Lipschitz condition is satisfied. This condition, together with continuity of the dependent variables in their respective intervals, assures us of existence of a unique solution satisfying the differential equations and the auxiliary conditions given at the lower end.

Uniqueness of the solution satisfying the conditions imposed at the lower end does not guarantee that there exists only one set of values of R_0 and Y_0 for a given set of turn speed and turn radius of the towing ship. In fact, Huang[2] reported a full-scale experiment in which the towline sometimes assumed different equilibrium configurations with the towplane flying at exactly the same altitude, speed, and turn radius. Skop and Choo[4] pursued this matter in depth for aerial towed systems. They found that, with other parameters held fixed, there exist critical values of the turn speed and the towline length above which multiple solutions exist. More specifically, there exists only one solution for the value of the speed (or the length) below the critical value regardless of whether the value of the length (or the speed) is above the critical value. Computations made in the present study seem to indicate that there exist only single solutions for the marine applications, in contrast with the findings in the aerial systems. This may be due to the speeds of ships being much smaller than those of airplanes and the water density being much larger than the air density.

Some Computational Results

In the last section, the towing problem is completely formulated except that the force $\mathbf{F}_B$ in Eq. (36) must be specified for the given body. Evaluation of $\mathbf{F}_B$ requires a considerable knowledge about the dynamical and the hydrodynamic characteristics of the towed body, including exact point of cable attachment. Our attention is focused on the spherical bodies because their characteristics are well known.

For a spherical body, the force at the point of the attachment is the same as that at the center of the sphere because no hydrodynamic moments exist (see Fig. 4). The force $\mathbf{F}_B$ on the towed sphere consists of its weight $-\mathbf{J}m_Bg$, buoyancy $\mathbf{J}\rho\bar{V}_B$ where $\bar{V}_B$ is the volume of the sphere, added-mass effect[14] $\mathbf{e}_R\frac{1}{2}\rho\bar{V}_BR\Omega^2$, and the drag $-\mathbf{e}_\theta C_{DB}A_B\frac{1}{2}\rho V_0^2$. Thus, Eq. (36) becomes, after rearrangement,

$$-\mathbf{e}_RM_BR\Omega^2 = \boldsymbol{\tau}T - \mathbf{J}w_B - \mathbf{e}_\theta C_{DB}A_B\tfrac{1}{2}\rho V_0^2 \quad \text{at} \quad s = 0 \tag{42}$$

where

$$M_B = m_B + \tfrac{1}{2}\rho\bar{V}_B \tag{43}$$

is the virtual mass of the towed sphere, and

$$w_B = m_Bg - \rho\bar{V}_Bg \tag{44}$$

In the form of the components in the directions of the natural coordinates, Eq. (42) is equivalent to

$$\tan\psi = -w_B/(M_BR\Omega^2) \quad \text{at} \quad s = 0 \tag{45}$$

$$\tan\phi = (w_B\sin\psi - M_BR\Omega^2\cos\psi)/(C_{DB}A_B\tfrac{1}{2}\rho R^2\Omega^2) \quad \text{at} \quad s = 0 \tag{46}$$

$$T = w_B\sin\psi\sin\phi - M_BR\Omega^2\cos\psi\sin\phi + C_{DB}A_B\tfrac{1}{2}\rho R^2\Omega^2\cos\phi \quad \text{at} \quad s = 0 \tag{47}$$

Thus, our problem is to solve the six nonlinear, ordinary differential equations (30–35) for the six variables T,ϕ,ψ,R,θ,Y, subject to the six initial conditions (37–39) and (45–47). Computation has been performed on a PDP-10 Computer, and the results are presented in Figs. 5–9.

Figures 5–7 show results which lead to the following possible conclusions for underwater applications. When a small turn radius of the towed body is needed, the ship should make either a fast turn or a tight turn. When a large depth of the towed body is needed, the ship should make either a slow turn or a tight turn. If both a small turn radius and a large depth of the towed body are required, then the ship has to make a tight turn. Though we can obtain this condition by using longer towline, this is certainly the least desirable alternative.

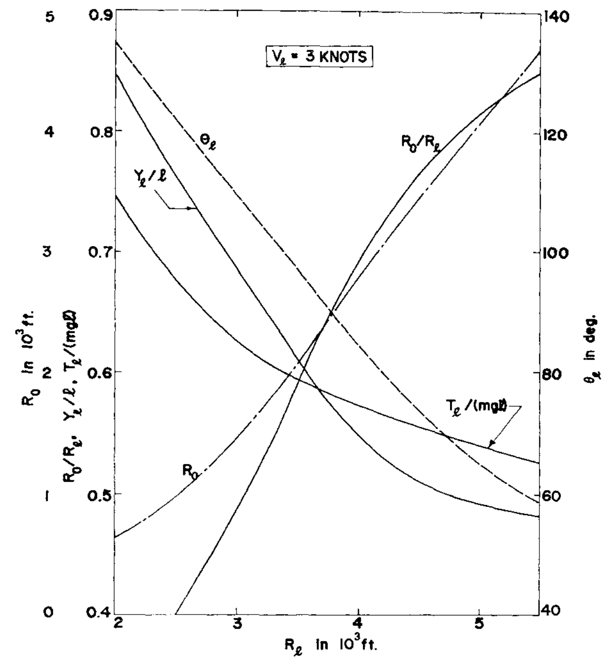

Fig. 6 Effect of the turn radius of the ship with its turn speed fixed. Water temp. 50° F; Circular cable, d = 0.5 in., mg = 0.3 lbf/ft., l = 6000 ft.; d_B = 1.2 ft., m_Bg = 250 lbf.

Due to the limitations of maneuverability and maximum speed of towing ships, as compared with those of aircraft, stationary or extremely small turn radius of the towed body may not be obtained in ordinary marine applications unless the body is extremely heavy or the towline is very long. However, this is possible in aerial systems as shown by Skop and Choo.[4]

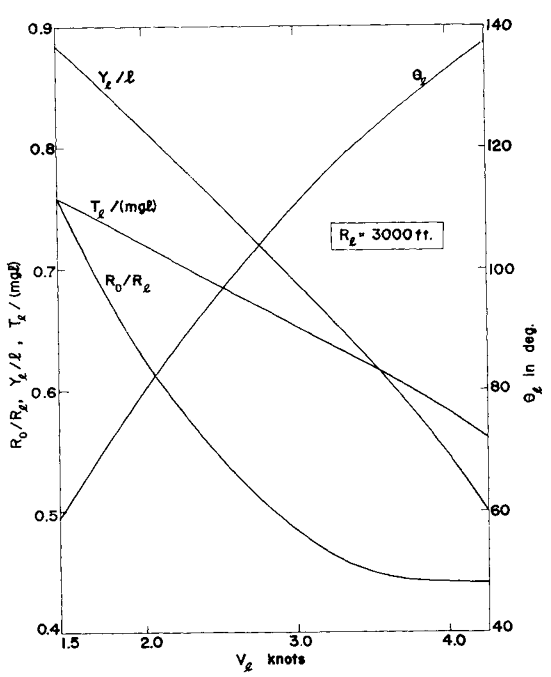

Fig. 5 Effect of the turn speed of the ship with its turn radius fixed. Water temp. 50° F; Circular cable, d = 0.5 in., mg = 0.3 lbf/ft., l = 6000 ft.; d_B = 1.2 ft., m_Bg = 250 lbf.

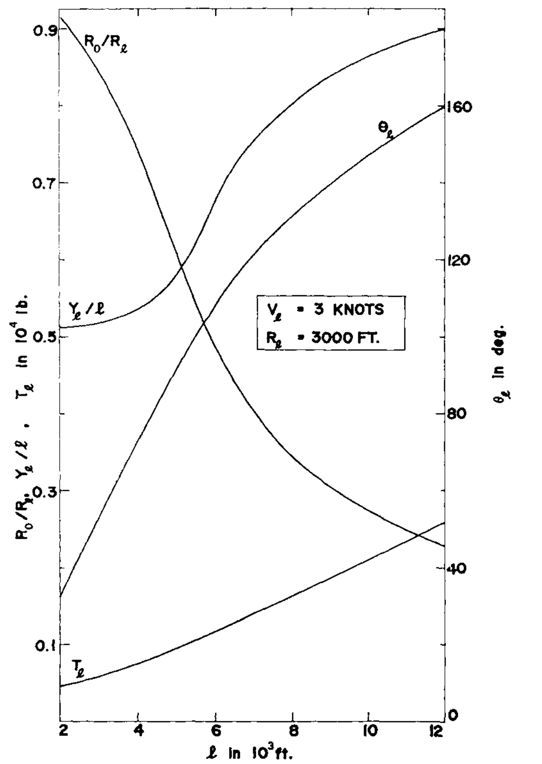

Fig. 7 Effect of the towline length. Water temp. 50° F; Circular cable, d = 0.5 in., mg = 0.3 lbf/ft.; d_B = 1.2 ft., m_Bg = 250 lbf.

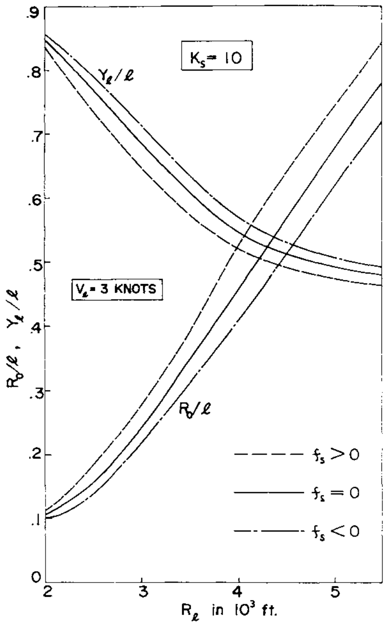

Fig. 8 Effect of the side force of stranded cables. Water temp. 50° F; d = 0.5 in., mg = 0.3 lb$_f$/ft., l = 6000 ft.; d_B = 1.2 ft., m_Bg = 250 lb$_f$.

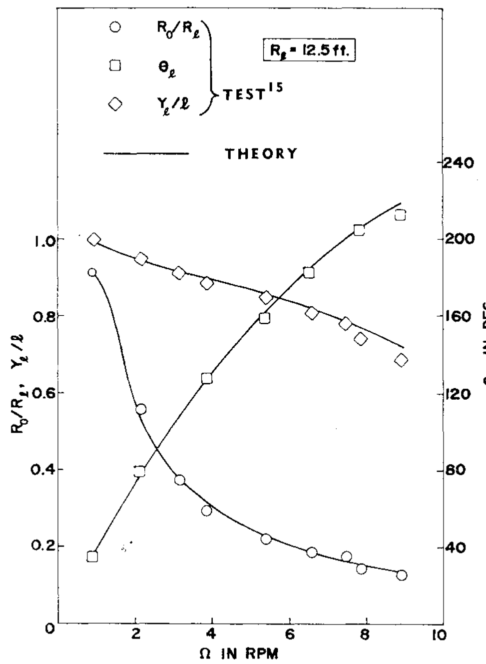

Fig. 9 Comparison of the present theory with the test of Noonan.[5] Water temp. 73°F; Stranded cable, 7 × 19, right-handed, d = 0.25 in., mg = 0.1072 lb$_f$/ft., l = 34 ft.; d_B = 5.0 in., m_Bg = 3.57 lb$_f$.

The side force of a stranded cable causes undesirable kite of the towed system on a straight path. This fact implies that, in circular towing, we may be able to use this property of stranded cables to our advantage. Indeed, Fig. 8 shows that a larger depth and a noticeably smaller turn radius of the body can be obtained when the ship equipped with a left-handed (or right-handed) stranded cable makes a turn such that the center of the towing circle remains to her port (or starboard).

Figure 9 demonstrates that the present analytical model is generally in good agreement with the laboratory tests of Noonan[15] for the cases of stranded cables used as towlines. But, he noticed that the model gives rather poor results for nylon towlines that undergo transverse oscillations induced by the fluid vortices shed behind them. The basic question related to this phenomenon is under what conditions the vortex shedding causes a towline to oscillate to such an extent that the hydrodynamic forces on the towline are noticeably affected. It requires intensive experimental work to answer this question.

Summary

Six equations governing the steady-state configuration of a towline connected to a towing vehicle moving on a circular path have been formulated. These six equations, of the type of nonlinear, ordinary differential equations, are derived on the assumption that the towline is an inextensible, bare cable of zero bending rigidity. The towline was treated as a continuum rather than as a set of lumped masses. The present analytical model is applicable for both aerial and underwater towed systems. Three of these equations are the equations of motion of a towline element, yielding the tension and two angles representing local orientation of towline element at a material point on the towline. The other three equations are geometrical relations for the three coordinates denoting the relative location of the material point.

The auxiliary conditions to be specified at the upper end are the turning radius and speed of the towing vehicle. At the lower end, the location, the tension, and the angles of orientation must be specified by means of the translational and the rotational equations of motion of the towed body. This means that, in general, the dynamical and the hydrodynamic characteristics of the towed body, including the point of cable attachment, must be completely specified.

A numerical method has been used for solving the governing differential equations, and some computational results have been obtained for the case of undersea towing of spherical bodies. At least in the ranges of the governing parameters for which the computations have been done, the following statements can be made about underwater towing:

1) A small turn radius of the towed body is obtained by fast or tight turn; a large depth of the towed body is obtained by slow or tight turn; both a small turn radius and a large depth of a body is obtained by a tight turn.

2) Side force of a left-handed (or right-handed) stranded cable causes the towed body to take a larger depth and a smaller turn radius when the ship turns such that the axis of rotation remains to her port (or starboard).

3) Multiple solutions were not encountered in undersea applications.

The model is generally in good agreement with the limited data available from laboratory tests. Further evaluation of the applicability of the model will be possible when full-scale test data are available.

Two related problems which require further studies have been identified. First, there exists an urgent need for systematic measurements of the hydrodynamic forces acting on stranded cables. Second, an analytical method must be developed that can predict when the vortex shedding causes the towline to oscillate to such an extent that its hydrodynamic forces are noticeably affected.

References

[1] Kline, J., "Final Report of Concepts for Airborne and Tethered VLF Antenna-Supporting Aerial Vehicles," CM-2520-B-2, Jan. 1968, Cornell Aeronautical Lab., Inc., Buffalo, N. Y.

[2] Hunag, J., "Mathematical Model for Long Cable Towed by Orbiting Aircraft," NADC-AM-6849, June 1969, U. S. Naval Air Development Center, Johnsville, Pa.

[3] Skop, R. A., "On the Shape of A Cable Towed in a Circular Path," NRL Rept. 7048, April 1970, Naval Research Lab., Washington, D. C.

[4] Skop, R. A. and Choo, Y., "The Configuration of a Cable Towed in a Circular Path," *Proceedings of the Symposium on Ocean Engineering*, Univ. of Pennsylvania, 1970, pp. 3E1–3E24.

[5] Crist, S. A., "Analysis of the Motion of a Long Wire Towed from an Orbiting Aircraft," *The Shock and Vibration Bulletin*, No. 41-Pt. 6, Dec. 1970, Naval Research Lab., Washington, D. C., pp. 61–73.

[6] Choo, Y., "Analytical Investigation of the Three-Dimensional Motion of Towed Systems," D. Eng. thesis, 1970, Catholic Univ. of America, Washington, D. C.

[7] Relf, E. F. and Powell, C. H., "Tests on Smooth and Stranded Wires Inclined to the Wind Direction and Comparison of Results on Stranded Wires in Air and Water," Rept. and Memo. 307, 1917, Aeronautical Research Committee, London, England.

[8] Gibbons, T. and Walton, C. O., "Evaluation of Two Methods for Predicting Towline Tensions and Configurations of a Towed Body System Using Bare Cable," Rept. 2313, Dec. 1966, David Taylor Model Basin, Washington, D. C.

[9] Hoerner, S. F., "*Fluid Dynamic Drag*," published by the author, 1965, Brick Town, N. J.

[10] Long, M. E., "Wind-Tunnel Tests of Mine Sweeper Cables," R-213, Dec. 1949, David Taylor Model Basin, Washington, D. C.

[11] Pode, L., "An Experimental Investigation of the Hydrodynamic Forces on Stranded Cables," Rept. 713, May 1950, David Taylor Model Basin, Washington, D. C.

[12] Schultz, M. P., "Wind-Tunnel Determination of the Aerodynamic Characteristics of Several Twisted Wire Ropes," Rept. 1645, June 1962, David Taylor Model Basin, Washington, D. C.

[13] Gay, S. M., "New Engineering Techniques for Applications to Deep-Water Mooring," ASME Paper 66-PET-31, 1966.

[14] Kochin, N. E., Kibel, I. A., and Roze, N. V., *Theoretical Hydromechanics*, Interscience, New York, 1965, pp. 329–332, 401–403.

[15] Noonan, B. J., "A Study of the Motion of a Cable System Towed from an Orbiting Vehicle," Ph.D. dissertation, 1971, Catholic Univ. of America, Washington, D. C.

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